Discourse Actions to Promote **Student Access**

Teachers at all grade levels can use these examples to support accessibility to high-quality mathematics.

Amber G. Candela, Melissa D. Boston, and Juli K. Dixon

Imagine a mathematics lesson during which a teacher is helping students develop an understanding of area as multiplication. (A video of a related scenario is included in Dixon, Brooks, and Carli 2019). The teacher has given each student square tiles and a handout with a rectangle on it. The students' rectangles range in area from 15 to 24 square units, but each student has only 12 square tiles and a dry erase board and marker. Students are covering their rectangles with tiles and beginning to determine that the number of tiles is insufficient to completely cover their rectangles.

Student: I don't have enough tiles to cover my rectangle.
Teacher: [*Acknowledging that the student's observation is correct*] Who has a strategy to share that doesn't involve getting more squares to help find the area?
Alex: You could multiply.

How would you respond to Alex? It matters what the teacher says next because different ways of following up on students' responses create different opportunities for students' learning (Herbel-Eisenmann and Breyfogle 2005). Consider the two different scenarios for responding to Alex (see figure 1). How did the teacher follow up on students' responses in each scenario? What types of opportunities for learning mathematics were provided in each scenario?

In this article, we highlight ways of following up on students' responses that have the potential to support students as they engage in cognitively challenging mathematical work and thinking, and we provide tools to identify and improve the use of these actions

Fig. 1

(a)

Scenario 1

Teacher: What would you multiply?
Alex: I would multiply these tiles by these tiles [*pointing to the tiles on the length of each side*].
Teacher: Nice thinking!

The teacher goes on to give students a page of examples with other rectangles with dimensions of the rectangles provided. Students work in pairs to find the areas.

(b)

Scenario 2

Teacher: What would you multiply?
Alex: I would multiply these tiles by these tiles [*pointing to the tiles on the length of each side*].
Teacher: Alex said she multiplied the side lengths. Alex, why did you choose to multiply?
Alex: Because we don't have enough tiles to just count them, so we need to multiply.
Teacher: [*Addressing other members of the class*] Why do you think Alex chose to multiply the side lengths? Dani?
Dani: Because we can see how many tiles fit in one row of the rectangle, and that is like one side of the rectangle. Then we can see how many rows we could fit in the entire rectangle, and that is like the other side.
Teacher: What did you get as the area, Alex?
Alex: [I got] 20 square units.
Teacher: How did you know the answer was 20?
Alex: I just knew, but since I had four rows of five, I could have done $5 + 5 + 5 + 5 = 20$.
Teacher: Class, turn and talk to someone next to you and discuss what this reminds you of in past mathematics discussions.

The class goes on to discuss having worked with arrays. The teacher then directs students to work in pairs to write an explanation and justification for why the area of a rectangle can be found by multiplying the dimensions of the rectangle.

Different ways of following up on students' responses create different opportunities for students' learning. Compare the teacher's responses to Alex in (a) scenario 1 with the teacher's responses in (b) scenario 2.

Amber G. Candela, candelaa@umsl.edu (@amcan36), is an assistant professor of mathematics education at the University of Missouri–St. Louis. She enjoys providing professional development to teachers and supporting each and every student's access to high-quality mathematics.

Melissa D. Boston, bostonm@duq.edu (@MBostonMath), is a professor of mathematics education at Duquesne University in Pittsburgh, Pennsylvania. She is interested in supporting mathematics teachers at all grade levels to use classroom observation tools to reflect on and enhance their practice.

Juli K. Dixon, juli.dixon@ucf.edu (@thestrokeofluck), is a professor of mathematics education at the University of Central Florida in Orlando. She is focused on improving teachers' mathematics knowledge for teaching so they can support their students' ability to communicate and justify mathematical ideas.

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In practice, identifying press actions in general is more important than correctly categorizing the type of press action.

in classrooms. Building on a rich history of research on the importance of discourse in mathematics classrooms (e.g., Lampert 2001; Michaels et al. 2010; NCTM 2014; O'Connor and Michaels 1996; Smith and Stein 2018), we identify *discourse actions* that generate or extend mathematical discourse during a mathematics lesson. (For a more detailed and comprehensive discussion of discourse actions, see Boston, Candela, and Dixon 2019). Using written vignettes from elementary, middle, and high school classrooms, we illustrate how specific discourse actions, entitled *linking* and *press*, can be used to follow up on students' responses. We encourage you to engage with all of the written vignettes. Even if the mathematical ideas in a vignette do not apply to the grades in which you teach, you will find it valuable to consider how each teacher uses discourse actions to provide opportunities for students to engage in mathematical activity and discourse. At the heart of the article (and our work) is a set of checklists and rubrics for measuring, analyzing, reflecting on, and improving teachers' (and students') use of discourse actions in the classroom. To supply you with opportunities to use the checklists and rubrics, we present videos of elementary, middle, and high school classrooms from the National Council of Teachers of Mathematics (NCTM) Principles to Actions Toolkit. We close the article by suggesting ways of using the checklists and rubrics to analyze and improve the use of discourse actions in your own practice.

DISCOURSE ACTIONS

Discourse actions include what a teacher or student says or does to elicit student contributions about a mathematical idea and generate ongoing discussion around student contributions (Boston, Candela, and Dixon 2019). In other words, discourse actions are actions taken to promote mathematical discourse. Discourse actions from teachers include such verbal prompts as asking questions, pressing students to say more about their initial response, and encouraging students to make links to their classmates' ideas. Discourse actions can also include other actions teachers take to promote discourse, such as using wait time or initiating a turn-and-talk. Discourse actions by students include providing mathematically valid explanations or linking to the ideas of their peers. These actions have been identified as productive by many mathematics education experts (e.g., NCTM 2014) and labeled with such terms as *talk moves* (Chapin, O'Connor, and Anderson 2013; Michaels and O'Connor 2015), *discourse moves* (Herbel-Eisenmann, Steele, and Cirillo 2013), and *Accountable Talk*™ (Michaels et al. 2010). Our contribution to this rich body of research on classroom discourse is the creation of tools to guide and empower teachers' use of discourse actions in the classroom. We use the term *discourse actions* intentionally, to indicate that teachers are making purposeful decisions about what they will say or do to promote discourse and make learning more public in the classroom.

Discourse actions provide access to learners by allowing processing time and providing scaffolding "just in time" (Dixon, Brooks, and Carli 2019) through questions asked by the teacher and ways in which the teacher follows up on contributions from students. Through discourse actions, a mathematical idea or strategy stays in the public space and is the object of discussion for a longer period of time as more students make contributions that deepen or extend the idea or strategy. Students who do not make contributions benefit by hearing additional explanations and ways of thinking. All students have more time to process the mathematics before the conversation moves to a new idea.

To follow up on students' responses in ways that elicit deeper thinking and invite other students into the discussion, we highlight the specific discourse actions of *press* and *linking* by the teacher and *providing* and *linking* by students. In the sections that follow, we explain and illustrate "teacher's press" and "students' providing" as well as "teacher's and students' linking."

Teacher's Press and Students' Providing

A teacher may decide to follow up on students' contributions by asking students to say more about their ideas and strategies. We label these discourse actions as *teacher's press* (Boston, Candela, and Dixon 2019; based on the work of Michaels et al. 2010). These actions include the following:

1. **Prompting students to explain their answer, strategy, or thinking**, to say more about *what* they did to solve the problem (e.g., from figure 1b, "What would you multiply?")
2. **Eliciting students to provide justification or mathematical proof** or indicate *why* what the student did was mathematically valid (e.g., from figure 1b, "Alex, why did you choose to multiply?")
3. **Asking students to validate the mathematical accuracy of their statements or claims** (e.g., from figure 1b, "How did you know the answer was 20?")

Teacher's press provides students with the opportunity to explain their thinking and provides the teacher and class with deeper insight into students' thinking. Through teacher's press, teachers ask students to explain, justify, or validate their contributions, allowing class members to hear the reasoning behind a mathematical idea or strategy offered by one of their peers. Keeping a mathematical idea or strategy at the center of the discussion for a longer period of time also affords more students the opportunity to come to understand the idea or strategy. Teachers can press students to provide mathematical justification or proof for their ideas, strategies, or claims. Through this discourse action, teachers support the classroom norm that students need to justify their solutions and ideas in mathematically valid ways. Once students have offered solutions, teachers can press by asking students to validate the mathematical accuracy of their ideas or claims, thus positioning students as mathematical authorities in the classroom. Asking students to validate computations and facts supports the classroom norm of students providing accurate responses and knowing why those responses are accurate.

Teacher's press actions create opportunities for students' providing actions, in which students explain their thinking, provide justifications, and validate mathematical accuracy. Students' providing actions occur when students give mathematical explanations or justifications without press from the teacher. Students' providing actions allow the teacher to hear their mathematical thinking and reasoning. Keeping a mathematical idea at the center of the discussion longer affords other students more opportunities to understand the mathematics.

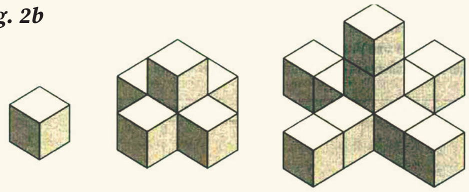
Consider the teacher's press and students' providing actions that occur in the vignette (see figure 2a) based on the middle school classroom of Mr. Dubno, featured in NCTM's Principles to Action Toolkit. Imagine this fictional vignette occurred prior to the video featured in the toolkit. Dubno provided students working in groups with the Counting Cubes task (see figure 2b). Students have been working on the task for approximately five minutes when Dubno approaches a group of four students to check on their progress. Read the vignette and identify any press actions made by the teacher or providing actions by students. We provide a blank, downloadable checklist, (see appendix A, available as a supplemental file online), in the online resources for you to capture evidence of the teacher's press and

Fig. 2a

Mr. Dubno: Where are you with this task?
Student 1: We think we know the pattern, and we are working on writing an expression.
Mr. Dubno: Tell me more.
Student 2: We noticed that each building has five more blocks than the one before it, so we think we will need to add five in our expression; but we are trying to figure out what we add it to.
Student 3: Yeah, we know we need a variable but, like, if we make n the building number, then adding five to the building number doesn't make sense.
Mr. Dubno: Why doesn't it make sense?
Student 1: Because building 3 would be 3 + 5, or 8 cubes, and we know that building 3 has 11 cubes because we counted.
Student 4: Oh, wait; we don't add five; we multiply five!
Mr. Dubno: Why do you think you should multiply rather than add?

Imagine that (a) this fictional vignette occurred before the video featured in NCTM's Principles to Action Toolkit (available to NCTM members—you must log in—at https://www.nctm.org/Conferences-and-Professional-Development/Principles-to-Actions-Toolkit/The-Case-of-Peter-Dubno-and-the-Counting-Cubes-Task/). The toolkit includes a full-page version of (b) the task on the next page.

Fig. 2b



1. Describe a pattern you see in the cube buildings.

2. Use your pattern to write an expression for the number of cubes in the *n*th building.

3. Use your expression to find the number of cubes in the fifth building. Check your results by constructing the fifth building and counting the cubes.

4. Look for a different pattern in the buildings. Describe the pattern and use it to write a different expression for the number of cubes in the *n*th building.

Adapted from "Counting Cubes," Lappan, Fey, Fitzgerald, Friel, and Phillips (2004). *Connected Mathematics™, Say It with Symbols: Algebraic Reasoning* [Teacher's Edition]. Glenview, IL: Pearson Prentice Hall. © Michigan State University.

Counting Cubes Task

students' providing during a mathematics lesson. Compare your responses with a colleague's if possible.

Using the checklist, we identify the press actions (see table 1) from the Counting Cubes vignette in figure 2a. We organize press actions into the three categories as a way of illustrating each one; in practice, however, identifying press actions in general is more important than correctly categorizing the type of press action. However, if teachers notice over time that they are not using press actions in particular categories, they could be intentional about planning to incorporate specific types of press actions.

Teacher's Linking and Student's Linking

Teacher's linking (Boston, Candela, and Dixon 2019; based on the work of Michaels et al. 2010) is a discourse action a teacher can use to provide opportunities for students to make connections, or links, to mathematical ideas, strategies, or explanations offered by their peers. Through teacher's linking, a teacher may ask students to restate or extend another student's contribution or to compare and contrast or relate another student's contribution. We identify three types of teacher's linking actions (Boston, Candela, and Dixon 2019; based on the work of Michaels et al. 2010):

1. **Revoicing students' contributions** (originally described by O'Connor and Michaels 1996); (e.g., from figure 1b, "Alex said she multiplied the side lengths.")
2. **Prompting students to take up the ideas of their peers** (e.g., from figure 1b, "Why do you think Alex chose to multiply the side lengths?")
3. **Focusing attention on students' explanations** (e.g., from figure 1b, "Class, turn and talk to someone next to you and discuss what this reminds you of in past mathematics discussions.")

Through revoicing, a teacher can repeat or rephrase a student's contribution to intentionally emphasize or clarify what the student was saying or to interject mathematical terminology. Teacher's linking occurs when teachers prompt students to make sense of or build on the ideas of their peers. This linking action encourages students to add on to or agree or disagree with their peers' mathematical ideas. By using these discourse actions, teachers give students ways to make direct links with their peers and build on the mathematical ideas of the lesson. Focusing attention on students' explanations directs students' attention toward a particularly salient mathematical contribution of another student and thereby helps keep everyone in the classroom together on the same mathematical concept. These linking actions highlight important mathematical ideas in students' explanations, and they prompt students to focus on key mathematical concepts within their peer' explanations.

Table 1 Examples of Press Actions from the Counting Cubes Task

Press Action	Evidence of Teacher's Press	Evidence of Students' Providing	Discussion
Prompting students to explain their answer, strategy, or thinking (say more about what he or she did to solve the problem)	"Tell me more."	"We noticed that each building had five more blocks than the one before it, so we think we will need to add five in our expression, but we are trying to figure out what we add it to."	Mr. Dubno continues to ask students to "say more" about their thinking and strategy. This is early in the lesson, so Mr. Dubno does not interject ideas or offer evaluation of students' work. Instead, Mr. Dubno presses their contributions to support them to articulate their thinking and engage with the mathematical ideas in the task.
Eliciting students to provide justification or mathematical proof (indicate why what the student did was mathematically valid)	"Why do you think you should multiply rather than add?"		
Asking students to validate their mathematical accuracy	"Why doesn't it make sense?"	"Because building 3 would be 3 plus 5, or 8 cubes, and we know that building 3 has 11 cubes because we counted."	

Students' linking occurs when students pick up and use ideas from their peers, such as explaining, relating to, or building on an idea offered by another student. Teachers can create opportunities for students to make connections to other students' ideas through teacher's linking actions, and students themselves can initiate students' linking without prompting from the teacher. Sometimes students will explicitly identify the other student and the idea he or she is building on. In scenario 2 of the opening vignette (see figure 1b), Dani might have replied, "Because we can see how many tiles fit in one row of the rectangle and that is like one side of the rectangle *just like how Alex was showing us*," thereby explicitly referring to Alex, the original speaker, and to Alex's idea. More common, as illustrated in Dani's actual response, students implicitly link to or build on the ideas of their peers without identifying the original speaker. Instances of students' linking indicate to the teacher that students are following the mathematical discourse and making sense of the mathematical ideas of their peers. Students' linking actions support the learning of other students by establishing (and making public) a coherent chain of ideas during mathematical discussions.

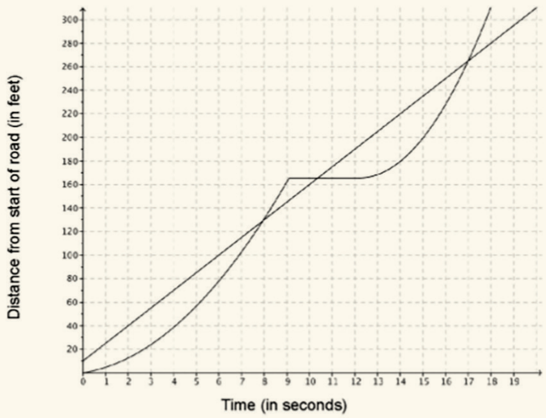
Consider the teacher's and students' linking actions that occur in figure 3, a vignette from Ms. Shackelford's high school algebra classroom featured on NCTM's Principles to Actions Toolkit. Shackelford's students were working on the Bike and Truck task (see figure 3a) when she noticed many students were incorrectly interpreting the flat (horizontal) portion of the graph of $K(t)$ between 9 and 12 seconds as the truck traveling in a straight path or along a flat surface. Shackelford decided to introduce this idea as coming from an "imaginary friend Chris" to provide an opportunity for students to discuss what is represented by the horizontal line on the distance-versus-time graph. The vignette (see figure 3b) is based on the actual discussion that occurred in Shackelford's classroom (see "Transcript of Video Clip 1," lines 1–40; available at https://www.nctm.org/Conferences-and-Professional-Development/Principles-to-Actions-Toolkit/The-Case-of-Shalunda-Shackelford-and-the-Bike-and-Truck-Task). Read the vignette and identify any linking actions made by the teacher or students. (Download appendix B, a blank checklist for linking actions.) Compare your responses with a colleague's if possible.

Fig. 3

(a)

Bike and Truck

A bicycle traveling at a steady rate and a truck are moving along a road in the same direction. The graph below shows their positions as a function of time. Let B(t) represent the bicycle's distance and K(t) represent the truck's distance.



1. Label the graphs appropriately with $B(t)$ and $K(t)$. Explain how you made your decision.
2. Describe the movement of the truck. Explain how you used the values of $B(t)$ and $K(t)$ to make decisions about your description.
3. Which vehicle was first to reach 300 feet from the start of the road? How can you use the domain and /or range to determine which vehicle was the first to reach 300 feet? Explain your reasoning in words.
4. Jack claims that the average rate of change for both the bicycle and the truck was the same in the first 17 seconds of travel. Explain why you agree or disagree with Jack.

Note: Some inconsistencies exist in how the graphs model the real-life movement of a bike and truck (e.g., a vehicle would not come to an immediate stop at 9 seconds). A graph that more realistically depicts the movement of a bike and truck is available at http://www.nctm.org/PtA/.

(b)

Teacher: We know we're talking about the truck. But he got to this point right here, and Chris was like, "Oh, that means that the truck was moving along in a straight path at that time." Do you agree? Don't say anything out loud, let everybody get a chance. Do you agree or disagree with Chris? Who agrees with Chris? [*show of hands*] Who disagrees with him? [*show of hands*] Make sure you justify your reasoning. If you disagree, say something. All right, go ahead, Jacobi.

Jacobi: OK, so, I agree; because, as you can see, he was not going in the constant rate right here [*pointing to the graph of K(t) from 0–8 seconds*]. Then he stopped [*pointing to the graph of K(t) at 9 seconds*]. I can see that he stopped and then [*moving his finger along the horizontal line of K(t) between 9 and 12 seconds*]—and then he was going real straight, so that's a constant rate. If you go straight and you don't have no, no, you know, curve or anything, then you're going up a straight path. But then he came back up [*pointing to the graph of K(t) after 12 seconds*]. He came back up, and he started, started back without a constant rate.

Teacher: OK. So, you are saying the truck is driving on a straight path, and so, he was going at a constant rate. All right, so Jacobi, step to the side because it's Charles's turn. Charles, did you hear what he had to say?

Charles: Um hmm.

Teacher: All right, do you still disagree with him?

Charles: OK, I disagree because when he stopped [*pointing to the graph of K(t) at 9 seconds*], the distance went, like, he didn't keep going because, like, time is still going, like he's not moving no more; and then he probably sped up when he came right there [*moving his finger along the graph of K(t) between 9 seconds and 12 seconds and after 12 seconds*].

Teacher: All right. Charles is saying the truck is not moving and then began to speed up.

Jacobi: I disagree.

Teacher: You still disagree with him? So, Jacobi, I need you to be looking at the graph. Charles, say it one more time.

Charles: OK. When he came up here, he stopped [*pointing to K(t) at 9 seconds*]. Time is still going, because he's not moving no more. So, the distance isn't going nowhere. And then he speeds up because he's moving again.

The (a) Bike and Truck Task is available at https://www.nctm.org/Conferences-and-Professional-Development/Principles-to-Actions-Toolkit /The-Case-of-Shalunda-Shackelford-and-the-Bike-and-Truck-Task/. The (b) vignette is based on the actual conversation in the classroom.

Table 2 Examples of Linking Actions from the Bike and Truck Vignette

Linking Action	Evidence of Teacher's Linking	Evidence of Students' Linking	Discussion
Revoicing students' contributions	"So, you are saying the truck is driving on a straight path, and so, he was going at a constant rate." "Charles is saying the truck is not moving and then began to speed up."		Ms. Shackelford emphasizes what Jacobi and Charles are saying to highlight each student's point because their different interpretations of the horizontal portion of the graph $K(t)$ are at the heart of the mathematical ideas and common errors that Ms. Shackelford wants to arise during the discussion.
Prompting students to take up the ideas of their peers	"Do you agree? Do you agree or disagree with Chris?" "If you disagree, say something." "All right, do you still disagree with him?"	"OK, so, I agree; because, as you can see, he was not going in the constant rate right here." "OK, I disagree because when he stopped . . ."	Ms. Shackelford frames the discussion by asking who agrees or disagrees with the mathematical ideas offered by the "imaginary friend Chris" and continues to ask Jacobi and Charles whether they agree or disagree with each other's ideas.
Focusing attention on students' explanations	"Make sure you justify your reasoning. If you disagree, say something. All right, go ahead, Jacobi." "Charles, did you hear what he had to say?" "Jacobi, you should be looking at the graph."		At the beginning of the vignette, Ms. Shackelford directs everyone's attention to the speakers. Throughout the discussion, she prompts Jacobi and Charles to listen to each other.

Using the checklist, we identify the linking actions (see table 2) in the Bike and Truck vignette. We categorize the linking actions to illustrate examples of each category, but in practice, identifying linking actions in general is more important than correctly categorizing the type of linking action. The checklist also allows for the collection of evidence over time, enabling teachers to identify the types of linking actions they are using regularly and those they may wish to incorporate more frequently.

USING DISCOURSE ACTIONS TO PROMOTE ACCESS AND EQUITY

We have described the discourse actions of teacher's press, teacher's linking, students' providing, and students' linking, and we have discussed how these actions can support student engagement and access. In addition to promoting greater access, discourse actions can create more equitable learning environments, as described in the essential element of Access and Equity in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014), by supporting the development of students' positive mathematical identity and agency (Berry and Ellis 2013; Martin 2012). When the teacher uses a linking action that includes a student's name and highlights the student's mathematical idea, the teacher assigns mathematical competence (Cohen et al. 1999) to the student and identifies the student as a mathematical authority. Similarly, by pressing a student to provide mathematical explanations, justification, and knowledge, the teacher attributes mathematical authority and agency to the student (Boaler and Greeno 2000; Boaler and Staples 2008). Discourse actions communicate to students that their mathematical ideas and strategies are valid and worthwhile and that they are capable of authoring important mathematical ideas.

Teacher's press allows students to present more thorough explanations of their thinking, so the teacher and students in the classroom community are able to better understand students' ideas. Prompting students to explain their thinking provides access to more learners because teachers can use press as a form of just-in-time scaffolding (Dixon, Brooks, and Carli 2019) to support students' conceptual understanding of the topic. Using the discourse action of teacher's press, teachers create opportunities for students to explain their thinking and for the teacher to understand the students' thinking. By encouraging students to provide more thorough explanations, teacher's press supports English Language Learners' (ELLs') mathematical learning in the classroom (de Araujo et al. 2018). Similarly, revoicing is a means for the teacher to model mathematical language and support ELLs in how to incorporate academic language into their mathematical explanations (Banse et al. 2016). For example, in figure 1b, Alex says, "I would multiply these tiles by these tiles," and points to the tiles on each side of the rectangle. The teacher clarifies by saying, "Alex said she multiplied the side lengths." Hence, through the action of revoicing, teachers can provide additional opportunities or access for students to engage with the mathematical contributions of their peers. These discourse actions offer chances for each and every student to make ideas public, and they provide a space where the expectation is for students to share not only their solutions but also their explanations and justifications.

IDENTIFYING DISCOURSE ACTIONS IN THE CLASSROOM

Using the opening vignette from an elementary school classroom, the Counting Cubes lesson from a middle school classroom, and the Bike and Truck lesson from a high school classroom, we intended to illustrate how the discourse actions of linking and press apply across PK–12 mathematical content and classrooms. As an opportunity to identify press and linking actions during a mathematics lesson, we suggest watching a video from an elementary, middle, or high school classroom (see table 3) featured on NCTM's Principles to Action Toolkit.

With the online resources, we offer blank versions of the checklists for press and linking featured in tables 1 and 2 as tools to identify the discourse actions of linking and press during mathematics lessons (see appendix A and appendix B). The checklists offer a way to keep track of teachers' and students' press and linking actions and to capture these discourse actions for reflection and discussion.

The Principles to Actions Toolkit offers a transcript for each video that may be helpful in identifying discourse actions. Use the checklists to record the instances of teacher's press and teacher's linking, and students' providing and students' linking, that you identify in the lesson. Discuss your responses with at least one other colleague. Did you identify the same discourse actions? How did the discourse actions you identified serve to support and extend mathematical discourse? How did the discourse actions promote access and equity? Once you have had a chance to debrief with a colleague, compare your responses and ideas to those provided with the online resources for this article. (*Note:* We identify discourse actions using line numbers from the transcripts provided on the NCTM Principles to Actions Toolkit.)

After collecting evidence of press and linking actions during a mathematics lesson, rubrics from a classroom observation instrument originally designed for research (Boston 2012, see table 4) can be used to monitor the use of discourse actions in a mathematics lesson over time (see appendix C, available as a supplemental file online). Essentially, data from the checklists are used to provide a rating on the rubrics for teacher's press, teacher's linking, students' providing, and students' linking. The quantity and quality of discourse actions determine the rating for each rubric. In general, at least three instances of a discourse action are necessary for a high rating, with the discourse actions generating a conceptually based explanation (for press and providing) or an explicit connection between students' ideas (for linking). The rubrics also contain very specific language within each score level. In this way, a teacher can determine how to make small changes to enhance the use of discourse actions during mathematics instruction.

We have also created a comprehensive set of resources, including additional checklists and rubrics, to support teachers' use of tasks, questioning, and discourse actions (see Boston, Candela, and Dixon 2019). Once you have used the checklist to gather evidence from the video lessons, use the rubrics to rate

Table 3 Practice Videos from NCTM's Principles to Actions Toolkit

Grade Level	Video	Link
PK–5	The Case of Katherine Casey and the Multiplication Strings Task	https://www.nctm.org/Conferences-and-Professional-Development/Principles-to-Actions-Toolkit/The-Case-of-Katherine-Casey-and-the-Multiplication-Strings-Task/
6–8	The Case of Peter Dubno and the Counting Cubes Task	https://www.nctm.org/Conferences-and-Professional-Development/Principles-to-Actions-Toolkit/The-Case-of-Peter-Dubno-and-the-Counting-Cubes-Task/
9–12	The Case of Shalunda Shackelford and the Bike and Truck Task, Clip 2	https://www.nctm.org/Conferences-and-Professional-Development/Principles-to-Actions-Toolkit/The-Case-of-Shalunda-Shackelford-and-the-Bike-and-Truck-Task/

Table 4 Teacher's Press Rubric (Boston, Candela, and Dixon 2019; adapted from Boston 2012)

Score	Descriptors for Levels of Press
4	The teacher consistently (almost always) asks students to provide evidence for their contributions by pressing for conceptual explanations or to explain their reasoning. There are few, if any, instances of missed press, in which the teacher needed to press and did not.
3	At least twice during the lesson, the teacher asks students to provide evidence for their contributions by pressing for conceptual explanations or for students to explain their reasoning. The teacher sometimes presses for explanations, but instances of missed press do occur.
2	Most of the press is for computational or procedural explanations or memorized knowledge, or there are one or more superficial, trivial, or formulaic efforts to ask students to provide evidence for their contributions or to explain their reasoning (for example, asking everyone, "How did you get that?" and then moving on without attending to student responses).
1	There are no efforts to ask students to provide evidence for their contributions, and there are no efforts to ask students to explain their thinking.
0	No class discussion, or class discussion is unrelated to mathematics.

the occurrence of press actions and linking actions in the video. Because the videos portray only portions of mathematics lessons, focus on identifying the types of discourse actions that would rate highly on the rubrics; particularly, whether the instances of press served to elicit conceptually based explanations and whether the instances of linking resulted in explicit connections between students' ideas. Discuss your rating with a colleague, including instances where the use of a discourse action supported students' mathematical learning and occasions where the use of a discourse action might have further enhanced students' opportunities for learning. What would be required (within the portion of the lesson featured in the video, or as an extension to the video) to advance to the next score level on the rubric?

CONNECTING TO PK–12 PRACTICE

The checklists and rubrics can be used to collect data from mathematics lessons and provide feedback to teachers (from instructional leaders, for peer-to-peer reflections, or for self-reflections) following live or video observations. An instructional coach can use these checklists during classroom observations and debriefing with teachers to provide feedback based on the evidence collected in the checklists. The checklists can be used in peer reflection, in which teachers observe one another (live or through video) and reflect together with other teachers, or in self-reflection, in which a teacher self-reflects with video. On the checklist, make notes of specifically what the teacher says as teacher's press or teacher's linking actions and what students say as students' providing or students' linking actions.
In either case, provide a space where teachers can reflect with a peer or a coach on (1) how the discourse actions supported students' learning; (2) places where discourse actions served to extend the conversation (thus providing additional opportunities for student access, engagement, and learning); and (3) places where incorporating more discourse actions would enhance student access, engagement, and learning.

Press actions hold students accountable to the mathematics, where the focus is on unpacking the mathematical ideas in the lesson. *Linking actions* hold students accountable to the classroom community.

The rubrics are most helpful for collecting data over time, for identifying strengths in teachers' use of discourse actions, and for identifying pathways for improvement. When used formatively, as a means of providing evidence to inform instructional change, the rubrics can indicate specific areas of focus or small changes to enhance the use of discourse actions in the classroom. Teachers who have used the rubrics in professional development noted that the rubrics (1) provided a framework for their own growth, where they could analyze their use of instructional strategies (such as using discourse actions) and determine how to improve; and (2) served as a framework to provide structured feedback to their peers, using wording directly from the rubrics (Candela 2016, 2019).

CONCLUSION

In summary, the discourse actions of press and linking support student engagement and access by providing teachers and students with a way to elicit student thinking. The discourse actions of press and linking create a space where students' thinking is made public and puts the mathematics in the hands of the students, rather than the teacher. This allows the students who are talking the opportunity to clarify their thinking and make sense of the mathematics. Discourse actions provide the opportunity for others in the class to hear the thinking of their peers, which allows them access to the discussion and supports their conceptual understanding of mathematics. In essence, press actions hold students accountable to the mathematics, where the focus is on unpacking the mathematical ideas in the lesson. Linking actions hold students accountable to the classroom community, where the focus is on the person who submitted the idea. The key is to make use of *actions* to support student *discourse* in mathematics.

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