
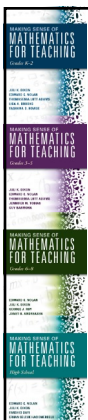


Making Sense of Mathematics for Teaching

Proportional Reasoning



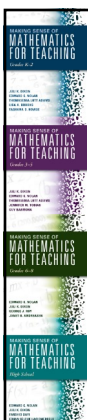
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Consider this . . .

An 8-foot tall tree is planted 3 feet from a street light. The tree casts a shadow from the light of the street light that is 12 feet long.

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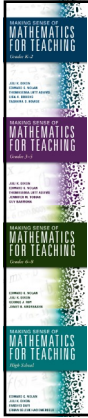
Three Reads

Support students to read a contextualized word problem three times:

1. What is the situation about?
2. What do the quantities describe?
3. What mathematical questions could you ask? (now compare)

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Content Connection

Grade 6
Solve problems using ratio reasoning.

Grade 7
Use proportional relationships to solve real-world and mathematical problems.

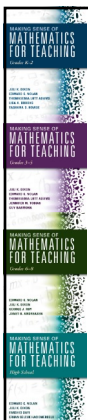
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Session Goals

- Understand the development of proportional reasoning in rigorous standards.
- Distinguish between proportions and proportional reasoning.
- Explore proportional reasoning in multiple contexts.

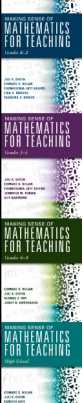
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Discourse Norms

- Provide explanations and justifications with solutions.
- Make sense of others' solutions.
- Communicate when you don't understand or don't agree.

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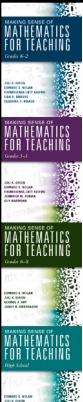


Definitions

Define a proportion.

Define proportional reasoning.

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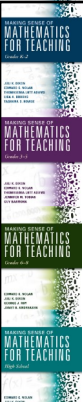
Consider this Task

Calvin joined a pool. The monthly fee is \$20 plus \$2 for each visit.

Calvin visited the pool 5 times in the first month and 10 times in the second month.

How much did Calvin pay each month?

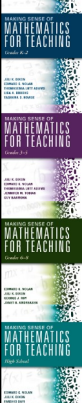
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Consider this Task

Shana drives to the store. She drives at 20 miles per hour. The journey takes 10 minutes.

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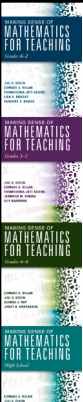
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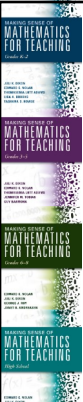


Consider this Task

Ivan mixes some pink paint. He uses four pints of white paint for every three pints of red paint. Ivan wants to mix more paint exactly the same color. He has $10 \frac{1}{2}$ pints of red paint.

How much white paint does he need?

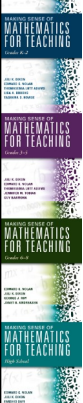
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When is it not proportional reasoning?

Provide an example of a scenario that does not involve proportional reasoning.

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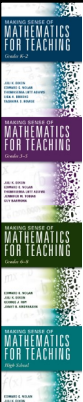


Plan with the TQE Process in Mind



- **Tasks** connect to learning goals and help identify student errors.
- **Questions** elicit mathematical understandings and common errors.
- **Evidence** drives scaffolding and guides extensions.

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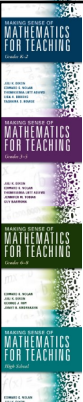


What is proportional reasoning?

In order to reason proportionally, students must not only reason with ratios, but they must also understand that the ratio of two quantities remains constant even as the corresponding values of the quantities change.

National Council of Teachers of Mathematics (2013).
Ratio and Proportion Research Brief.

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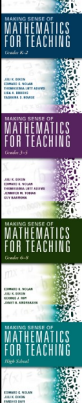


Solve ...

Proportion Problems

1. The table on a map shows 2 kilometers represents 25 miles. If a given measurement on the map is 24 kilometers, how many miles are represented?
2. Lauren and Kenneth purchased pencils. Lauren bought 10 pencils for \$3.50. Kenneth purchased 5 pencils for \$1.80. Who got the better deal?
3. Brandon is making paint to use in art class. Yesterday, she mixed white and red paint together. Today she used more red paint on the same amount of white paint to make her mixture. What do you say about the color of today's mixture compared to yesterday's mixture?
4. Heidi and Meredith decide to invest money in a 6% 401k plan. Heidi invests \$1,000, which is 6% of their total investment. How much do Heidi and Meredith invest together?
5. Angel is planning a party for 40 people. She decides to make a full punch that includes using 1/2 cups of orange juice for every 3 servings. If she wants to prepare 1 serving for each person, how much orange juice will she need?
6. Quinn drives at an average rate of 25 miles per hour for 3 hours to get to her friend's house. Returning to a take her 3 hrs. How long to return to Quinn's house?
7. Judy's cell phone service charges her \$4.50 per month for phone service. Last month she used 20 minutes of service. How much more service did she use this month if she paid \$18.50 for service?
8. Randy and Jorge both go to a run, when they run, both run at the same rate. Today they started at different times. Randy had run 5 miles when Jorge had run 2 miles. How many miles had Jorge run when Randy had run 8 miles?

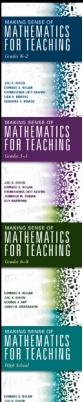
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Explore

The scale on a map is 2 centimeters represents 25 miles. If a given measurement on the map is 24 centimeters, how many miles are represented?

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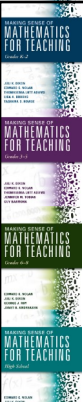


Explore

Randy and Jorge both go for a run. When they run, both run at the same rate. Today, they started at different times. Randy had run 3 miles when Jorge had run 2 miles.

How many miles had Jorge run when Randy had run 6 miles?

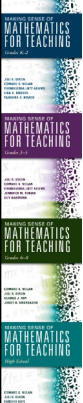
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Explore

Heidi and Meredith decide to invest money in a local ice cream shop. Heidi invests \$1,500, which is 60% of their total investment. How much do Heidi and Meredith invest together?

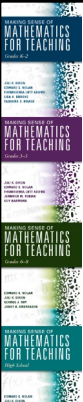
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Explore

Kimberly is making paint to use in art class. Yesterday, she mixed white and red paint together. Today, she used more red paint and the same amount of white paint to make her mixture. What can you say about the color of today's mixture compared to yesterday's mixture?

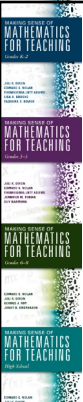
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Explore

Lauren and Kenneth purchased pencils. Lauren bought 10 pencils for \$3.50, and Kenneth purchased 5 pencils for \$1.80. Who got the better deal?

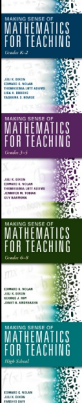
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Explore

Gloria drives at an average rate of 25 miles per hour for 3 hours to get to her friend's house. How long will it take her if she is able to average 50 miles per hour?

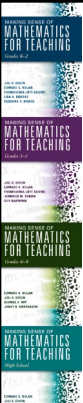
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Explore

Judy's cell phone service charges her \$14.50 per month for phone service, plus \$0.15 for each text she sends or receives. Last month, she sent or received 27 texts and her bill was \$18.55. How much will she pay if she sends or receives 54 texts this month?

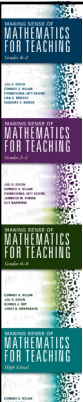
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Explore

Angel is planning a party for 60 people. She decides to make a fruit punch that includes using $1\frac{1}{2}$ pints of orange juice for every 8 servings. If she wants to prepare 1 serving for each person, how much orange juice will she need?

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Classify ...

Judy's cell phone service charges her \$14.50 per month for phone service, plus \$0.15 for each text she sends or receives. Last month, she sent or received 27 texts and her bill was \$18.55. How much will she pay if she sends or receives 54 texts this month?

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Angel is planning a party for 60 people. She decides to make a fruit punch that includes using $1\frac{1}{2}$ pints of orange juice for every 8 servings. If she wants to prepare 1 serving for each person, how much orange juice will she need?

Lauren and Kenneth purchased pencils. Lauren bought 10 pencils for \$3.50, and Kenneth purchased 5 pencils for \$1.80. Who got the better deal?

Meredith decide to invest in a local ice cream parlor. She invests \$1,500, which is a 10% investment. How much money will she have after 1 year?

The scale on a map is 2 centimeters represents 25 miles. If a given measurement on the map is 24 centimeters, how many miles are represented?


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Plan with the TQE Process in Mind



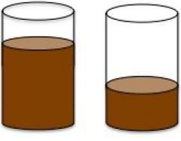
- Select appropriate **T**asks to support identified learning goals.
- Facilitate productive **Q**uestions to engage students in mathematical practices.
- Collect and use student **E**vidence in the formative assessment process.

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Chocolate Milk

Glass A and glass B contain chocolate milk that tastes the same.

Create a task for this situation appropriate for your students.



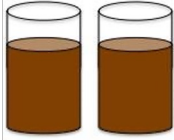
A B

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Chocolate Milk

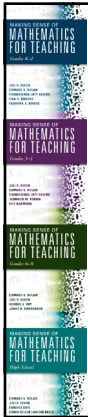
Glass B contains chocolate milk with a stronger chocolate taste than glass A.

Create a task for this situation appropriate for your students.



A B

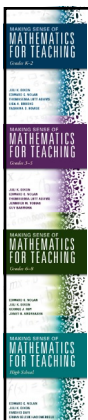
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How is proportional reasoning developed?

- Proportions should be addressed conceptually before procedurally.
- Emphasis should be placed on understanding what is happening in proportional situations.
- Conceptual explorations should be connected to procedures before procedures are practiced.

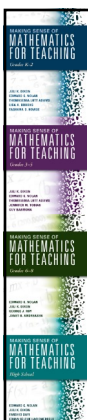
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What does this mean for you?

- Explore proportional reasoning in different contexts and with different structures.
- Become a critical consumer of proportional reasoning tasks.

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Session Goals

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- Distinguish between proportions and proportional reasoning.
- Explore proportional reasoning in multiple contexts.

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