

Multiplication Fact Fluency as a Foundation for Algebra

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Goals

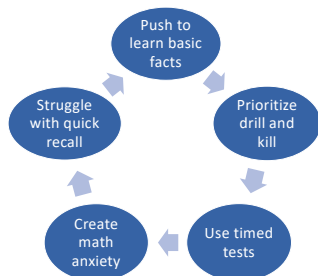


- Examine structures for developing procedural fluency grounded in multiplicative reasoning
- Make sense of the Six Fluency Tactics
- Explore how a strategic approach to develop automaticity prepares students for algebra and beyond

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What is the current focus of fluency instruction?



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Why is this a problem?

“Evidence strongly suggests that timed tests cause the early onset of math anxiety for students across the achievement range” (Boaler, 2014, p. 469).

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Why does this current practice miss the point?

Readiness for algebra requires “a grasp of the meaning of the basic operations of addition, subtraction, multiplication, and division. It must also include use of the commutative, associative, and distributive properties; computational facility; and the knowledge of how to apply the operations to problem solving” (NMAP, 2008, p. 17).

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What strategies might students use to determine the product of 6×7 if they did not know it?

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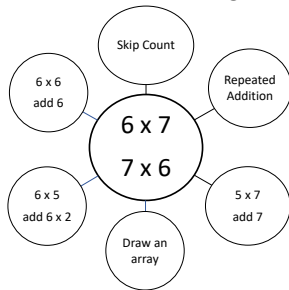
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What mathematics is involved in each strategy?

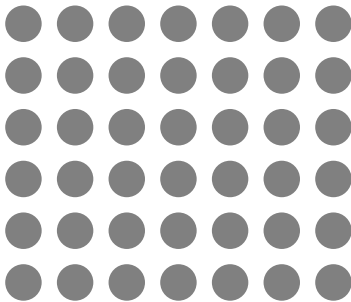
- Drawing
- Counting Strategies
- Multiplicative Reasoning

Label each strategy as:

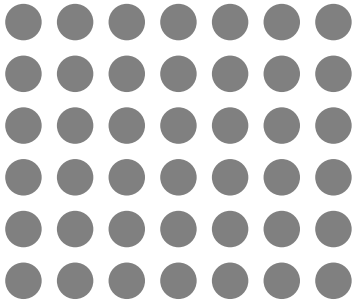
- Drawing,
- Counting Strategies, or
- Multiplicative Reasoning



6×7



$6 \times 7 = (6 \times 5) + (6 \times 2)$

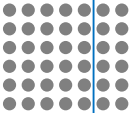


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How would we get to the strategies symbolically?




$6 \times 7 = (6 \times 5) + (6 \times 2)$

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6×7

Consider this video from *Go Math!*



Houghton Mifflin Harcourt

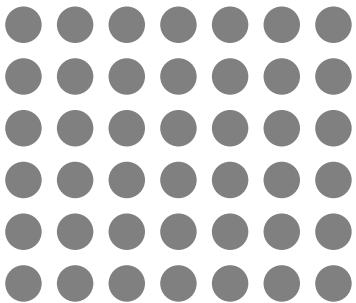
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6 x 7 as 3 x 7 and double

$$6 \times 7 = 2 \times (3 \times 7)$$



What is procedural fluency?

The seminal work of the National Research Council (NRC, 2001), *Adding It Up*, defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (p. 5).

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How does this position evolve into a fluency program?

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The Fact Tactics™ Fluency Program

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graph TD
    A[Build multiplicative reasoning skills.] --> B[Introduce facts in a strategic order.]
    B --> C[Practice facts through sense making.]
    C --> D[Challenge students who need more.]
    D --> E[Achieve automaticity.]
  
```

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What is automaticity?

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Automaticity is the ability to execute procedures without conscious thought (NRC, 2001). It can be thought of as *just knowing*.

We want this – but not first!

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The Six Fluency Tactics

- The Grounding Tactic
- The Linking Tactic
- The Strategic Repetition Tactic
- The Review Tactic
- The Assessment Tactic
- The Extension Tactic

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The Grounding Tactic

Students will be more likely to make sense of their basic facts if they have already developed an understanding of the meaning of multiplication and can apply that meaning to strategies for multiplying basic facts.

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The Grounding Tactic

It starts with word problems.

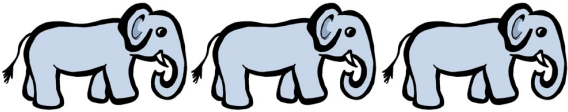
Write a word problem for 6×7 .

Is your word problem modeled by 6×7 or 7×6 ?

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Consider 3×4

I have 3 elephants. Each elephant has 4 legs. How many legs are there in all?



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How are we preparing for algebra?

What happens to the elephants when we get to algebra?

Consider $4n$

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The Grounding Tactic

Beyond making sense of multiplication, there are facts students need to know to make the most of Fact Tactics™.

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The Grounding Tactic

Prerequisites for the Fact Tactics Fluency Program:

- ✓ The Meaning of Multiplication
- ✓ Doubles
- ✓ Fives
- ✓ 3 x 3


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The Six Fluency Tactics

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The Linking Tactic

The order of facts in building fact fluency has the power to support reasoning and sense making. It is about being Fact Tactical 😊.

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The Linking Tactic

We do not start with one and zero facts...

The Linking Tactic

How many facts are we talking about anyway?

Twenty facts.

The Linking Tactic

X	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

The Linking Tactic

This progression addresses all the facts in 20 weeks and provides a tactical development of fact fluency.

- | | | | |
|---------|---------|---------|---------|
| ✓ 3 x 4 | ✓ 3 x 7 | ✓ 4 x 8 | ✓ 8 x 8 |
| ✓ 3 x 6 | ✓ 4 x 7 | ✓ 6 x 8 | ✓ 4 x 9 |
| ✓ 4 x 4 | ✓ 6 x 7 | ✓ 7 x 8 | ✓ 7 x 9 |
| ✓ 4 x 6 | ✓ 7 x 7 | ✓ 3 x 9 | ✓ 8 x 9 |
| ✓ 6 x 6 | ✓ 3 x 8 | ✓ 6 x 9 | ✓ 9 x 9 |

The Fact Tactics Web

$$3 \times 4$$

How might students derive this fact using multiplicative reasoning as they develop fact fluency? Create a sample web to support your thinking.

$$2 \times 4$$

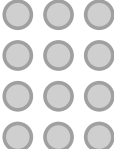
add 4

$$3 \times 4 = (2 \times 4) + 4$$

2 x 3
add 2 x 3

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4×3



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2 x 3
and
double

How is this different?

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$4 \times 3 = 2 \times (2 \times 3)$

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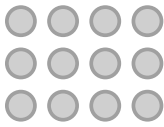
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3 x 3
add 3

How would you
make sense of this?

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$3 \times 4 = (3 \times 3) + 3$



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The Linking Tactic

Notice how previous facts are used to derive the new focus fact. This is linking.

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The Extension Tactic

Students who already “know” their basic facts, reason and connect concepts to procedures through extension tasks. These more complicated problems are solved by applying the same properties that are emphasized with the Fact Tactics™ Webs.

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The Extension Tactic: Supporting Students who Need More

$$3 \times 26$$

Consider students who already know the product of 3×6 without using a strategy.

The Extension Tactic

How does your new math knowledge for teaching influence how you “see” these extension tasks?

- ✓ 2×34
- ✓ 3×26
- ✓ 4×24

The Extension Tactic

What about these?

- ✓ $3(2x + 6)$
- ✓ $3(x - 4)$
- ✓ $(x + 2)(x + 3)$

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Components

- The Grounding Tactic
- The Linking Tactic
- The Strategic Repetition Tactic
- The Review Tactic
- The Assessment Tactic
- The Extension Tactic

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The Strategic Repetition Tactic

Students should be provided many opportunities to verbalize *both* their strategies and the product. As students verbalize their strategies, they will approach automaticity with the focus and partner facts.

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The Strategic Repetition Tactic

Examine the Fact Tactics™ Web for 6×7 and the partner fact 7×6 . Choose your favorite strategy and record it on the Fact Tactics™ Card.

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The Strategic Repetition Tactic

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The Strategic Repetition Tactic

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- **The Review Tactic**
- The Assessment Tactic
- The Extension Tactic

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The Review Tactic

Practice in the form of review is crucial for developing long term fact fluency. Review is strategic when it is connected to fact strategies.

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6 x 7 or 7 x 6
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4 x 7 or 7 x 4
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3 x 7 or 7 x 3
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6 x 6
#FactTactics™

4 x 4
#FactTactics™

3 x 6 or 6 x 3
#FactTactics™

4 x 6 or 6 x 4
#FactTactics™

3 x 4 or 4 x 3
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5 x 6 = 30
Add 2 x 6 = 12
Product: 42

2 x 3 = 14
Add double
Product: 28

4 x 6 = 24
Add 2 x 6 = 12
Product: 36

3 x 4 = 12
Add 4
Product: 16

4 x 3 = 12
Add 3 x 3 = 9
Product: 21

2 x 6 = 12
add 6
Product: 18

5 x 4 = 20
add 4 to get 24
Product: 24

3 x 3 = 9
Add 3
Product: 12

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The Assessment Tactic

Timed tests are not encouraged. Instead, students should be given five facts at a time as mini assessments. Students should indicate the product of each fact and state if they “just knew it” or provide the Fact Tactics™ strategy they used.

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The Assessment Tactic

Examine the mini assessments scheduled for weeks 1 and 2 when the focus facts are 3×4 and 3×6 . What do you notice about the quizzes? How might the results of the quizzes support a formative assessment process?

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The Assessment Tactic

Week One Assessment: Record the product of each fact. Indicate if you just knew it by checking the box, or describe the Fact Tactics™ strategy you used to determine it.

	I just knew it!	I used the Fact Tactics strategy of ...
1. $3 \times 4 =$ <input style="width: 50px;" type="text"/>	<input type="checkbox"/>	
2. $3 \times 3 =$ <input style="width: 50px;" type="text"/>	<input type="checkbox"/>	
3. $5 \times 3 =$ <input style="width: 50px;" type="text"/>	<input type="checkbox"/>	
4. $4 \times 3 =$ <input style="width: 50px;" type="text"/>	<input type="checkbox"/>	
5. $2 \times 6 =$ <input style="width: 50px;" type="text"/>	<input type="checkbox"/>	

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The Assessment Tactic

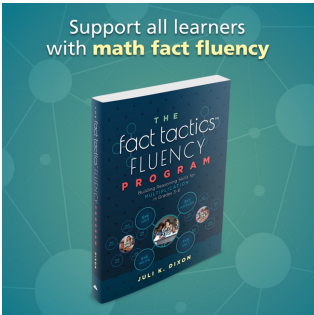

Week Two Assessment: Record the product of each fact. Indicate if you just knew it by checking the box, or describe the Fact Tactics™ strategy you used to determine it.

	I just knew it!	I used the Fact Tactics strategy of ...
1. $4 \times 3 =$ <input style="width: 50px;" type="text"/>	<input type="checkbox"/>	
2. $3 \times 6 =$ <input style="width: 50px;" type="text"/>	<input type="checkbox"/>	
3. $2 \times 4 =$ <input style="width: 50px;" type="text"/>	<input type="checkbox"/>	
4. $4 \times 5 =$ <input style="width: 50px;" type="text"/>	<input type="checkbox"/>	
5. $6 \times 3 =$ <input style="width: 50px;" type="text"/>	<input type="checkbox"/>	

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Support all learners
with **math fact fluency**

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